# Agricultural Supply Response: Adaptive- and Rational Expectations

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*This paper discusses the model of agricultural supply response. We contrast between traditional adaptive expectations model developed by Nerlove (1957) and the rational expectations model by Eckstein (1984). We then put emphasis on econometric aspect of modeling and testing the two approaches.*

## I. Nerlove's Model of Supply Response: Adaptive Expectation and Partial Adjustment

### 1.1. The Model

Nerlove claimed that farmer's planting decision depends on the prices they expected to receive when the crop was marketed. In turn, the actual price for the crop depends on the amount actually harvested as well as the current level of demand.

Nerlove model is basically characterized with both **adaptive expectations** and **partial adjustment**. The standard representation of the model is:

$$P_t^* = P_{t-1}^* + b\,(P_{t-1} - P_{t-1}^*) + u_t \ , \qquad 0 \le b \le 1 \tag{1}$$

$$X_t = X_{t-1} + g\,(X_t^* - X_{t-1}) + v_t \ , \qquad 0 \le g \tag{2}$$

$u$ and $v$ are random terms with zero expected values.

The first equation resembles the adaptive expectations. It says that, the expected price $P^*$ for this year is equal to the expected price last year plus the difference between the actual and the expected price last year, times the *expectation coefficient* $b$. The second equation, on the other hand, resembles the adjustment process. The quantity supplied this year is the same as the quantity supplied last year plus the difference between the expected (or, desired-) supplies this year and the actual supply last year, times the *adjustment coefficient* $g$. We can see this adjustment coefficient as the level of technology or the speed of adjustment. So, the farmer could not move to equilibrium instantaneously in the short-run.

In addition to the equations above, we have the supply response function as follows:

$$X_t^* = a + bP_t^* + cz_t + w_t \qquad (3)$$

where $z$ is other exogenous factor and $w$ is random term with zero expected value.
Allowing for continuing lags, we can rewrite (1) as

$$P_t^* = \sum_{s=1}^{\infty} b(1-b)^{s-1} \cdot P_{t-s} \qquad (4)[1]$$

We can now combine (3) and (4) as:

$$X_t^* = a + b\sum_{s=1}^{\infty} b(1-b)^{s-1} \cdot P_{t-s} + cz_t + w_t \qquad (5)$$

Now we have the following equation implies both adaptive expectations and partial adjustment process, by plugging (5) into (2):

$$X_t = ag + bg\sum_{s=1}^{\infty} b(1-b)^{s-1} \cdot P_{t-s} + cgz_t + gw_t + gX_{t-1} + X_{t-1} + v_t \qquad (6)$$

or, suppressing $g$ in the constant and error terms, we have:

$$X_t = a + bg\sum_{s=1}^{\infty} b(1-b)^{s-1} \cdot P_{t-s} + cgz_t + (1-g)X_{t-1} + u_t \qquad (7)$$

## 1.2. Estimation

For estimation purpose, we can substitute (1) and (3) into (2) and get:

$$X_t = abg + bbgP_{t-1} + [(1-b) + (1-g)]X_{t-1} + [-(1-b)(1-g)]X_{t-2} +$$
$$cgz_t + [-cg(1-b)]z_{t-1} + v_t - (1-b)v_{t-1} + gw_t - g(1-b)w_{t-1} + bgu_t \qquad (8)$$

or, in reduced form:

$$X_t = p_1 + p_2P_{t-1} + p_3X_{t-1} + p_4X_{t-2} + p_5z_t + p_6z_{t-1} + e \qquad (9)$$

where:

[1] Thus, the RHS is the “certainty equivalence” to the LHS.

$$p_1 = abg$$

$$p_2 = bbg$$

$$p_3 = (1 - b) + (1 - g)$$

$$p_4 = -(1 - b)(1 - g)$$

$$p_5 = cg$$

$$p_6 = -cg(1 - b)$$

$$e = v_t - (1 - b)v_{t-1} + gw_t - g(1 - b)w_{t-1} + bgu_t \qquad (10)$$

Obviously, the reduced form (9) is **overidentified**, since we have six $p$ 's with only five structural parameters ($a$, $b$, $c$, $b, g$). So, we impose a nonlinear constraint as follows (Sadoulet and de Janvry, 1995):

$$p_6^2 - p_4 p_5^2 + p_3 p_5 p_6 = 0 \qquad (11)$$

Then if we run a maximum-likelihood estimation on (9) we can next solve the structural coefficients as follows:

$$g^2 + (\hat{p}_3 - 2)g + 1 - \hat{p}_3 - \hat{p}_4 = 0$$

$$1 + \hat{p}_4 / (1 - g) = b$$

$$\hat{p}_1 / gb = a$$

$$\hat{p}_2 / gb = b$$

$$\hat{p}_3 / g = c \qquad (12)$$

The short-run price response is $\hat{p}_2$ (see eq. 9), and the long-run price response is $b$ (see eq. 3). It is obvious from (12) that $b > \hat{p}_2$ since both $b$ and $g$ are less than one.

*Result 1. In Nerlovian adaptive expectations-partial adjustment model, price response is greater in the long-run than in the short-run.*

### 1.3. Criticisms

Major critiques to Nerlove's model include the one (e.g. Sheffrin, 1996) pointing out that different assumptions concerning the formation of price expectation could dramatically alter the actual price dynamics in the market. If the price expectation is based on last year's price, there would be a potential for significant instability in prices and production. For example, a bad weather destroys part of the crop so that the price rises above normal. If farmers expect the price to prevail, they would plant more than they usually do. Then, when the resulting crop is harvested, prices will fall below normal. If this now-price is expected to prevail, the year after that will be characterized by lower output and higher prices. This oscillation could be growing or dampening overtime, depending on supply and demand elasticities.

Other criticizes that this model does not allow information about the causes of price movements or the probability of a future shock to influence estimates of future prices. In addition, adaptive expectations model implies that the distributed lag parameters are restricted in an ad hoc way, because the restrictions are not the result of an optimization process (Fisher, 1982). This use of lagged dependent variable in the partial adjustment mechanism can generate problem of non-stationarity. However, estimating the model using MLE as suggested above could yield consistent, asymptotically normal, and asymptotically efficient. For this to hold, we need to have a large sample. (Johnston and DiNardo, 1997).

### 1.4. Problems in Empirics

Inspired by Nerlove's model of supply response, there have been numerous empirical studies conducted. Askari and Cummings (1976, 1977) examined more than 600 estimates of supply response to price. The studies the surveyed mostly deal with annual food crops in developed countries as well as in South and East Asia. However, in their survey, Askari and Cummings uncovered some problems that were frequently encountered in empirical application of Nerlove's model.

First, what price. Price series most frequently used were either: the price of the crop actually received by farmers, the ratio of that price over some CPI, the ratio of that price over some input price index, or the ratio of that price to some index of competitive crops. The answer to this question is related to why a farmer would produce more of a crop. That is, either to increase his own consumption, or to keep it the same give a rise in input costs, to buy more of other goods, or to keep his consumption of other goods the same.

The second problem is related to the exogenous factors to consider (the *z* variable). Theoretically, this variable includes weather, land reform, irrigation, mechanization, credit, soil quality, etc. Nerlove suggested using weather (i.e. rainfall) and time trend (so, to capture technological progress). In practice, we may encounter too much rain, and also, the timing of rain itself can be an important factor affecting the supply.

The model does not consider the fact that the lag structure may vary from one type of crop to another. In general, we expect perennial crops to have longer lags than annual crops. This issue of lag structure becomes very important especially when we explain aggregate output.

## II. Eckstein's Model of Supply Response: Rational Expectation

Eckstein (1984) argues that rational farmers are unlikely to interpret price fluctuations that are serially uncorrelated as signaling a permanent alteration in the incentives confronting them. In addition, any permanent and transitory change affects the dynamic response of the cropped area. Therefore, predictions with respect to changes in policy require complete identification of the structural parameters that govern the production and the price processes.

Using rational expectation framework, Eckstein examined Egyptian data on cotton and wheat prices and production, 1913-1969. He found that as the relative prices of the crops change, cotton and wheat areas respond in opposite directions and fluctuate frequently toward the means.

### 2.1. The Model

Eckstein's model assumes a farmer maximizing the expected value of discounted profits by choosing cotton and wheat land allocations ($A_1$ and $A_2$), subject to technology and uncertain price movements. The farmer makes current decision based on the expectation of future prices, since the selling prices of the crops are unknown when input decisions are made.

$$\max E_{-1} \lim_{N\to\infty} \sum_{t=0}^{N} b^t (X_{1t} + \frac{P_{2t}}{P_{1t}} X_{2t}) \quad (13)^2$$

$$\text{s.t. } A_{1t} + A_{2t} = \overline{A} \quad (14)$$

$$X_{1t} = (f_1 + f_{1t} - \frac{g_1}{2} A_{1t}) A_{1t} + d_1 (1 - \frac{A_{1t-1}}{\overline{A}} - \frac{A_{1t}}{\overline{A}}) A_{1t} \quad (15)$$

$$X_{2t} = (f_2 + f_{2t}) A_{2t} \quad (16)$$

$$\Omega_{t-1} = \{A_{1t-1}, A_{1t-2}, \ldots, f_{1t-1}, f_{1t-2}, \ldots, \frac{P_{2t-1}}{P_{1t-1}}, \frac{P_{2t-2}}{P_{1t-2}}, \ldots, S_{t-1}, S_{t-2}, \ldots\} \quad (17),$$

where $X_{it}$ is production of crop $i$ at time $t$, $P$ is price, and $A_{it}$ is land allocated at time $t$-1 for producing crop $i$ at time $t$. $\overline{A}$ is the total available land and $f_{it}$ is a shock to the production of crop $i$ at time $t$. $\Omega_t$ is information set available at time $t$ and expectation is defined as $E_t(X) = E(X \mid \Omega_t)$. $S$ indicates exogenous factors other than price. The production function for cotton is assumed quadratic while the production function for wheat is linear (thus, closed to Nerlove's model). The term $f_1$, $f_2$, $g_1$, and $d_1$ are positive scalars. The second term in (15) implies that cultivating cotton deteriorates land productivity directly. Implicitly assumed above is that both crops can be produced on the same plot at the same period.

The system can also be represented as:

$$\max\ E_{-1} \lim_{N\to\infty} \sum_{t=0}^{N} b^t \left[ (f_1 + f_{1t}) A_{1t} - \frac{g_1}{2} A_{1t}^2 + \frac{d_1}{\overline{A}} (\overline{A} - A_{1t-1} - A_{1t}) - R_t A_{1t} + R_t \overline{A} \right] \quad (18)$$

which is subject to the law of motion of

$$d(L) Z_t = U_t \quad (19)$$

where Z'$_t$ = $(f_{1t}, R_t, S_t)$, $S$ is a vector of $n$-2 exogenous variables.

$E(U_t \mid \Omega_{t-1}) = 0$ and $E(U_t U_t^{'}) = \Sigma_t$, $\Sigma_t$ is a positive semidefinite matrix.

$R_t = \frac{P_{2t}}{P_{1t}} (f_2 + f_{2t})$ is the real shadow price for land allocated to cotton.

Ekcstein derived the Euler equation from (18) as:

[2] So prices are normalized by $P_{1t}$

$$A_{1t} = l_1 A_{1t-1} - \frac{l_1 \overline{A}}{d_1} \sum_{j=0}^{\infty} (b\, l_1)^j [f_1 + d_1 + E_{t-1}(f_{1t+j}) - E_{t-1}(R_{t+j}) \qquad (20)$$

where $l_1$ is a function of $g_1, d_1, \overline{A},$ and $b$ .

If $d_1 > 0$, $-1 < l_1 < 0$ , we have:

$$\frac{\partial A_{1t}}{\partial E_{t-1}(R_t)} < 0 \quad \text{and} \quad \frac{\partial A_{1t}}{\partial E_{t-1}(R_{t+1})} > 0 \qquad (21)^3$$

Thus, in equlibrium, as the expected current price of cotton relative to wheat increases, more land is allocated to cotton. However, if next year's price of cotton relative to wheat is expected to increase, then the quality of current land allocated to cotton will decrease. This equilibrium results create an oscillation of cotton and wheat land allocation following a price shock.

Following Hansen and Sargent (1980), Eckstein rewrote the decision rule as an exact closed-form analytical solution:

$$A_{1t} = l_1 A_{1t-1} + g + m_1(L) f_{1t-1} + m_2(L) R_{t-1} + m_3(L) S_{1t-1} + m_3(L) S_{2t-1} + \ldots + m_n(L) S_{n-2t-1} \qquad (22)$$

where

$$g = \frac{l_1 \overline{A}(f_1 + d_1)}{d_1 b (1 - b\, l_1)} \quad \text{and} \quad m_1(L) = m_{i0} + m_{i1} L + \ldots + m_{1J} L^J \qquad (23)$$

Since the shocks and the relative price are serially uncorrelated and are independent of variables in the information set, then $f_{1t}$ has zero mean and $R_t$ has positive mean

$$A_{1t} = l_1 A_{1t-1} + \frac{l_1 \overline{A}}{d_1 (1 - l_1)} \cdot \overline{R} \qquad (24)$$

$$A_1^* = \frac{g}{1 - l_1} + \frac{l_1 \overline{A}}{d_1 (1 - l_1 b)(1 - l_1)} \cdot \overline{R} \qquad (25)$$

[3] In case of "adjustment cost", $d_1 < 0$, $0 < l_1 < 1$, we have same sign for first term but opposite sign for second term.

where $\overline{R}$ and $A_1^*$ are the mean price and the mean of $A_{1t}$, respectively. These two equations imply that farmers do not interpret price fluctuation and production shock that are serially uncorrelated as the signals for permanent alteration.

We can also see from (24) and (25) that:

$$d_1 > 0 \quad : \qquad \frac{\partial A_{1t}}{\partial d_1} < 0 \ , \ \frac{\partial A_1^*}{\partial d_1} < 0 \ \text{ and } \ \frac{\partial A_{1t}}{\partial d_1} > \frac{\partial A_1^*}{\partial d_1} \tag{26}$$

That is, an increase in the rate of land deterioration decreases the area allocated to cotton. And, the short-run effect is greater than the long-run effect, as opposed to Nerlove's pattern. However, in the case of adjustment costs ($d_1 < 0$), the short-run effect is less than the long-run effect as in Nerlove.

*Result 2. In rational expectations model, the structure of the stochastic process of the relative price is critical to the movements of land allocations due to changes in prices or other variables affecting them. Therefore, supply response depends on technology and the parameters of the price processes.*

## 2.2. Estimation

Eckstein claimed that, even though the reduced form of (22) is observationally equivalent to Nerlove's (9), the interpretations could be very different. This is due to the fact that Nerlove's model does not investigate jointly the dynamic of the production process and the dynamics of the actual prices. Ecstein's model, on the other hand, emphasizes the importance of technology dynamic structure, all information available, and the price movement over time.

Using maximum-likelihood estimation, Eckstein estimated jointly (22) and (19). The reduced form (see 29 below) yields a vector-autoregression with, after testing the data, three lags. Ecsktein assumed that:

$$R_t = a_0 + a_1 R_{t-1} + a_2 R_{t-2} + u_t^R \tag{27}$$

$$f_{1t} = r f_{1t-1} + u_t^q \tag{28}$$

with $\left| r \right| < 1$ and the roots of $\left| 1 - a_1 z - a_2 z^2 \right| = 0$ have modulus less than one. Then the VAR can be written as:

$$\begin{bmatrix} A_{1t} \\ R_t \end{bmatrix} = \begin{bmatrix} m_0 \\ a_0 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} D + \begin{bmatrix} r + l_1 & m_1 \\ 0 & a_1 \end{bmatrix} \begin{bmatrix} A_{1t-1} \\ R_{t-1} \end{bmatrix} +$$

$$\begin{bmatrix} -r l_1 & m_2 - r m_1 \\ 0 & a_2 \end{bmatrix} \begin{bmatrix} A_{1t-2} \\ R_{1t-2} \end{bmatrix} + \begin{bmatrix} 0 & -r m_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A_{1t-3} \\ R_{1t-3} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \tag{29}$$

where $D$ is dummy variable [4]. Cross-equation restrictions imply:

$$m_1 = \frac{l_1}{d} \left( \frac{a_1 + a_2 l}{1 - a_1 l - a_2 l} \right)$$

$$m_2 = \frac{l_1}{d} \left( \frac{a_2}{1 - a_1 l - a_2 l^2} \right)$$

$$m_3 = \frac{l_1}{d} \left( \frac{r}{1 - l r} \right)$$

$$e_{1t} = m_3 u^f_{t-1} \quad \text{and} \quad e_{1t} = u^R_t \tag{30}$$

where $d = d_1 / \overline{A}$. The innovation vector, $e_t$ is normally distributed with $E(e_t e_t') = V$. Since there are 11 nonzero regressors in (29) and there are only 9 free parameters ($l_1, r, m_0, a_0, a_1, a_2, w_1, w_2, d$), then (29) is **overidentified**

To check the plausibility of his rational expectations model as well as the overidentifying cross-equation restrictions, Ekcstein ran a joint estimation of the land-allocation-decision rule and the stochastic processes. He performed three LR tests of the model. First, he tested zero restrictions on $l_1, r, a_1, a_2, d$. Rejecting it, he concluded that land allocation Granger causes relative price. Second, he tested the cross-equation restriction while maintaining the zero restrictions. He failed to reject the model, implying a support for rational expectation model over adaptive expectation. Finally, he tested the cross-equation restrictions, without zero restriction and rejected the model. He argued that this rejection is due to zero restriction on price equations, not to rational expectations cross-equations. Therefore, he concluded, the rational expectations model sufficiently reproduces the oscillation in crop areas observed in the data.

[4] Eckstein used dummies for the Second World War, in which period the production of wheat and cotton were altered dramatically.

### 2.3. Criticisms

#### 2.3.1. Basic Assumption

Rational expectations hypothesis assumes that agents employ all information available to him. However, this is not always the case. Farmers may not use all information available, since it incurs some cost to acquire it. Or, they may have incomplete understanding of the mechanism of price determination. Finally, they may not know how to forecast the changes in exogenous variables. On the other hand, under rational expectations the forecast error must not only have zero mean but must also be unpredictable by any means. If the errors are predictable, they then are some relevant and available information and not using them is violating the rational expectations hypothesis. In reality, agents may learn overtime and to some extent are able to predict. Fortunately, a correct use of lagged variable may overcome such problem in specifying the model. That is, once future error is predictable, agents simply need to "reformulate their model" until it becomes unpredictable.

#### 2.3.2. Linearity Issue

The linearity in the whole system does not allow the model to represent the variables being studied conveniently. Hansen and Singleton (1982), on the other hand, suggest the use of linear-quadratic models (i.e. quadratic optimization problem subject to linear constraint). This form of linear-quadratic model is of advantage, as it could lead to restrictions on a system of constant coefficient linear difference equations, which in turn provide complete characterization of the equilibrium time path of the variables. In addition, the obtained Euler equation implies orthogonality condition that depends on variables observed by econometrician and unknown parameters chararacterizing the production. Also, the closed-form analytical solution (see 22) can be obtained without having to impose some strong assumptions on the stochastic properties of the "forcing variable", the nature of the production, or the technology (Hansen and Singleton, 1982).

If, otherwise, the dynamic model is nonlinear, then, as suggested by Hansen and Singleton, it is preferable to apply generalized instrumental variables estimation rather than maximum likelihood method of estimation as used by Eckstein. It has been recognized that maximum likelihood estimators are in general asymptotically more efficient than nonlinear instrumental variables, **if** the distributional assumptions are correctly specified. In other words, maximum likelihood estimation requires additional assumption about distribution, imposed on the model. On the other hand, generalized instrumental variables method does not require additional assumption, since it can work directly from orthogonality conditions implied by the Euler equation. Consequently, estimation and inference can be conducted when only a subset of the economic environment is specified *a priori*.

To apply generalized instrumental variables estimation to Eckstein model (given that it were specified as nonlinear dynamic), we consider again $E(U_t \mid \Omega_{t-1}) = 0$ above. Moreover, we can define a function:

$$f\ (R_{t+n}, S_t, f) = g\ (R_{t+n}, f\ ) \otimes S_t \quad (31)$$

where $\otimes$ is the Kronecker product. Then we can write:

$$E[f(R_{t+n}, S_t, f_0)] = 0 \quad (32)$$ [5]

Let $h_0(f) = E[f(R_{t+n}, S_t, f)]$, then the method of moments estimator of $h_0$ is:

$$h_T(f) = \tfrac{1}{T}\sum_{t=1}^{T} f(R_{t+n}, S_{t,} f) \quad (33)$$

Evaluating (33) at $f = f_0$ gives a value close to zero for very large *T*. Then we are left to minimize the function $J_T$ given by:

$$J_T(f) = h_t(f)' W_T h_t(f) \quad (31)$$

where $W_T$ is a positive definite matrix.

### 2.3.3. Testing Procedure

The three tests conducted by Eckstein are all likelihood ratio tests. In contrast, Goodwin and Sheffrin (1982) employed prediction test as well[6]. They argued that as long as there is more than

[5] This orthogonality condition is important. We have discussed above that rational expectations hypothesis assumes that the model has to include all the relevant variables. However, this is unlikely to be satisfied in practice. The question of how many variables are enough to get reliable estimates can be hard to answer. Fortunately, we can make approximation by using generalized method of moments to determine the conditions to be satisfied for large samples. (Johnston and DiNardo, 1997).

one underlying source of uncertainty in the model, a predictive equation based on rational expectations model should have a lower error variance than the one based on time-series model for the price. This is because some information is bst when different sources of uncertainty are aggregated into a single stochastic process. For accuracy purpose on the Eckstein model, we need to report this test in addition to the likelihood ratio tests.

## 3. Conclusion

We discuss the difference between the traditional Nerlove's supply response model with Ekcstein's model. The former is characterized by adaptive expectations and partial adjustment mechanism, while the latter is the application of rational expectations hypothesis.

We pointed out that in Nerlovian adaptive expectations-partial adjustment model, price response is greater in the long-run than in the short-run. Meanwhile, in rational expectations model, the structure of the stochastic process of the relative price is critical to the movements of land allocations due to changes in prices or other variables affecting them. Therefore, supply response depends on technology and the parameters of the price processes.

Some suggestions are provided, including the use of nonlinear (e.g. linear-quadratic) dynamic model that is less demanding on restricting the stochastic process. In addition we suggest additional test, prediction test that compares the error variance between predictive equation based on rational expectations model and predictive equation based on a time-series model for price. Further direction can be of the form of using ECM (error correction mechanism) rather than VAR model, testing rational expectations hypothesis in cointegrated VAR model, etc.

[6] They also had the third test for orthogonality. They argued that it should not be possible the forecasts implied by mo del with any information that was available at the time the forecast was made. To run this test, they needed a variable that potentially contains additional information about the future price. They then used the future market prices. If this test were to be conducted too in Eckstein model, we need to define such variable.

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