Economies of Scale: Replicating Christensen and Greene (1976)

by Arianto A. Patunru
Department of Economics, University of Indonesia
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1. Introduction

This exercise is based on a class assignment in University of Illinois, instructed by Prof. Carl Nelson. It is to replicate some of the results on economies of scale reported by Christensen and Greene (Economies of Scale in U.S. Electric Power Generation, *Journal of Political Economy*, 84:4, Part 1, 1976). Christensen and Green updated the data used by Nerlove (1955; *Returns to Scale in Electricity Supply*, reprinted in C.F. Christ, ed., *Measurement in Economics: Studies in Honor of Yehuda Grunfeld*, Stanford Univ. Press; 1963:167-98) to 1970. They found that by 1970 the bulk of electricity generation in the United States came from firms operating very near the bottom of their average cost curves. They concluded that a small number of extremely large firms are not required for efficient production and that policies promoting competition in electric power generation cannot be faulted in terms of sacrificing economies-of-scale.

2. Replicating Nerlove (1955)

First, we replicate some of the principal returns-to-scale results reported by Nerlove (1955). The equation estimated by Nerlove is:

$$ln(cp3) = \beta_0 + \beta_y\, ln(kwh) + \beta_1\, ln(p13) + \beta_2\, ln(p23)$$

where *ln* is natural logarithm and:

cp3	= relative total costs
kwh	= kilowatt-hours as measure of outputs
p13	= labor relative price with respect to fuel price
p23	= capital relative price with respect to fuel price

Nerlove (1963) reports parameter estimates for β_y , β_1 , and β_2 as 0.721(0.175), 0.562(0.198), and –0.003(0.192), respectively, where figures in parentheses are standard errors; and R^2 of 0.931.

2.1. Estimation

Our replication yields:

```
    Source |       SS       df       MS              Number of obs =     145
-----------+------------------------------           F(  3,   141) =  639.98
     Model |  294.667577     3  98.2225256           Prob > F      =  0.0000
  Residual |   21.640321   141  .153477454           R-squared     =  0.9316
-----------+------------------------------           Adj R-squared =  0.9301
     Total |  316.307898   144  2.19658262           Root MSE      =  .39176

------------------------------------------------------------------------------
     lncp3 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-----------+------------------------------------------------------------------
     lnkwh |   .7206875   .0174357    41.33   0.000     .6862183    .7551567
     lnp13 |   .5929097   .2045722     2.90   0.004     .1884845    .9973349
     lnp23 |  -.0073811   .1907356    -0.04   0.969    -.3844523    .3696901
     _cons |  -4.690789   .8848715    -5.30   0.000    -6.440119   -2.941459
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Therefore we cannot replicate Nerlove's results precisely. Both estimates and their standard errors are different from what Nerlove obtains. However, we obtain R^2 that is close to Nerlove's.

2.2. Returns-to-Scale

From the estimation above we obtain a 95% confidence interval[1] for $\hat{\beta}_y$ as follows:

$$\hat{\beta}_y \in [0.686, 0.755]$$

This implies that, at the 5% significance level, there is **no** constant returns-to-scale, since $\hat{\beta}_y = 1$ lies outside the interval.

Another way to examine constant returns-to-scale is by testing the hypothesis of $\beta_y = 1$ against the alternative, $\beta_y \neq 1$. The test statistic[2] for this is –16.0196. Comparing this to critical value of $Z_{\alpha/2} = \pm 1.96, \alpha = 0.05$ leads us to rejecting the null hypothesis. Again, we conclude there is **no** constant returns-to-scale.

[1] We can also calculate this manually using the formula:

$$CI_{\hat{\beta}_{y0.05}} = [\hat{\beta}_y \pm Z_{\alpha/2} \cdot \sigma_{\hat{\beta}_y}]$$

[2] The formula is

$\frac{\hat{\beta}_y - 1}{\sigma_{\hat{\beta}_y}}$; in addition we could also use F-test provided by STATA (see appendix), where we get F(1,141) = 256.63 with Prob > F = 0.000. Taking square root of this F-stat gives us the t-stat above.

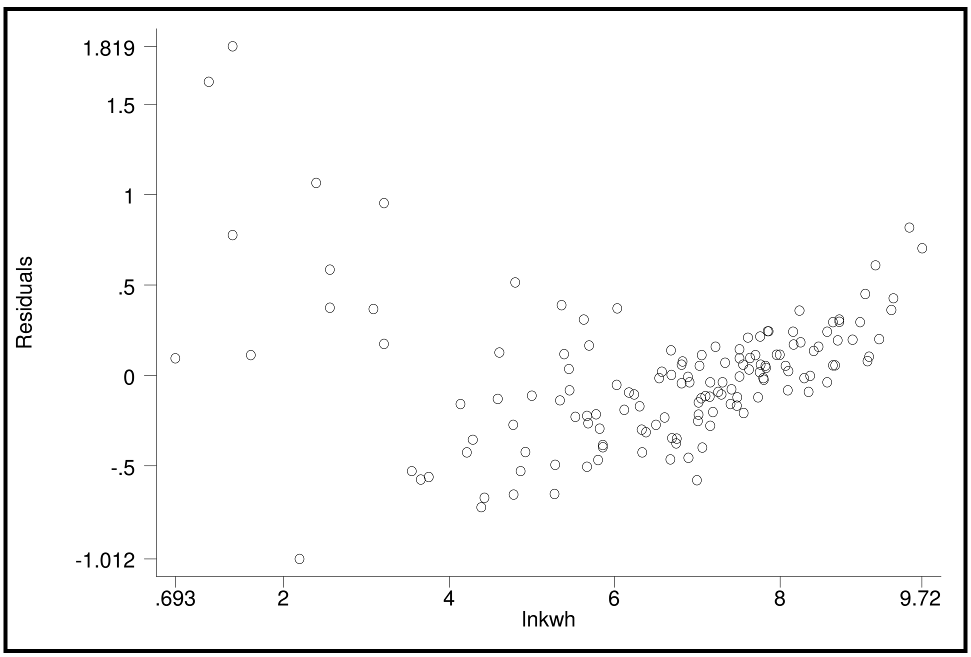
Now we are interested to know whether the returns-to-scale is actually increasing or decreasing. We use a point estimate r [3] with relation to β_y as $\beta_y \equiv 1/r$. Solving for r we have: $r \equiv 1/\hat{\beta}_y = 1.39$. By the property of Cobb-Douglas function, we conclude an **increasing** returns-to-scale. Lastly, we can obtain **positive** economies of scale: $e = r - 1 = 0.39$.

2.3. Factor Share

The demands for each factor of production will be positive only if the factor share, α_i is positive, i = 1, 2, 3. The factor shares are determined by $\alpha_i = \beta_i / \beta_y \approx \hat{\beta}_i \cdot r$. Therefore we obtain α_2 = -0.00738 x 1.39 = -0.01026. Using this result we can test the hypothesis that $\alpha_2 = 0$. This gives a test statistic of 0.0387 [4]. Thus, we cannot reject the hypothesis. We conclude that the factor share of capital is not significantly different from zero. This result is striking since it implies that capital is not being used as a production factor in electricity generation. This might be the result why Nerlove was unsatisfied.

2.4. Residuals

In the following figure we plot the residuals from the estimated regression against the output:



[3] r being the sum of factors' exponents in a Cobb-Douglas function (i.e. returns-to-scale parameter).

[4] The formula is

$$\frac{\alpha_2 - 0}{\sigma_{\alpha_2}}, \text{ where } \sigma_{\alpha_2} = \sqrt{\text{var}(r \cdot \hat{\beta}_2)} = r \cdot \sigma_{\hat{\beta}_2} = 0.265 \text{ ; and } p\text{-value is } 0.484.$$

In the figure we see that the residuals are positive at small levels of output, negative at medium levels, and positive again at larger level of outputs. This U-shaped pattern is close to what Nerlove found and it means that the regression results overestimate the true costs at very low and very high levels of output, and underestimate them at intermediate levels of output.

Finally we note that the sample correlation between the residuals and the output is zero. This is because in OLS estimation, the explanatory variables (here including the output) are orthogonal to the error term, i.e. they are independent.

3. Fixing the Data

Next we use UPDATE data provided by Berndt (*The Practice of Econometrics: Classic and Contemporary*, Addison-Wesley Publishing, 1991). The data consists of 9 columns with 99 observations. The columns are OBSNO (observation number), COST70 (total costs), KWH70 (output), PL70 (labor price), PK70 (capital price), and PF70 (fuel price). After examining the data we suspect that there is a decimal placing error in PL70 column. Summary statistics shows that the mean and standard deviation of PL70 are extremely high compared to those of PK70 and PF70:

Variable	Obs	Mean	Std. Dev.	Min	Max
pl70	99	7968.554	1256.624	5063.49	10806.2
pk70	99	72.34388	9.727865	39.127	92.063
pf70	99	30.3333	8.041643	9	50.4516

To fix this, we divide the PL70 entries by 100. We then obtain the following statistics:

Variable	Obs	Mean	Std. Dev.	Min	Max
pl70	99	79.68554	12.56624	50.6349	108.062
pk70	99	72.34388	9.727865	39.127	92.063
pf70	99	30.3333	8.041643	9	50.4516

which is more comparable.

4. Cobb-Douglas Cost Function

After transforming the data into relative prices with respect to fuel price we construct a Cobb-Douglas cost function as follows:

$$(COST / PF) = \beta_0 (KWH)^{\beta_Y} (PL / PF)^{\beta_L} (PK / PF)^{\beta_K}$$

or, in their log forms:

$$\ln c = \beta_0 + \beta_Y \ln y + \beta_L \ln pl + \beta_K \ln pk$$

where c : COST/PF
y : KWH
pl : PL/PF
pk : PK/PF
and ln means natural logarithm.

We obtain the following results from estimation:

```
      Source |       SS       df       MS              Number of obs =      99
-------------+------------------------------           F(  3,    95) = 1440.98
       Model |  192.449487     3  64.1498291           Prob > F      =  0.0000
    Residual |  4.22922446    95  .044518152           R-squared     =  0.9785
-------------+------------------------------           Adj R-squared =  0.9778
       Total |  196.678712    98  2.00692563           Root MSE      =  .21099

------------------------------------------------------------------------------
         lnc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lny |   .8253341   .0125666    65.68   0.000     .8003862    .8502819
        lnpl |   .1452104   .1090331     1.33   0.186    -.0712477    .3616685
        lnpk |    .164958   .0980337     1.68   0.096    -.0296635    .3595795
       _cons |  -7.423647    .132391   -56.07   0.000    -7.686477   -7.160818
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The table shows that both ln pl and ln pk (i.e. ln(PL/PF) and ln(PK/PF), resp.) are not significant at the 5 percent level, i.e. the t-statistics are 1.33 and 1.68, respectively, with p-value of 0.186 and 0.096.

We can calculate the derived value of the coefficient for ln(PF) as follows:

$$\ln(COST) - \ln(PF) = \beta_0 + \beta_Y \ln(KWH) + \beta_L [\ln(PL) - \ln(PF)] + \beta_K [\ln(PK) - \ln(PF)]$$

$$\ln(COST) = \beta_0 + \beta_Y \ln(KWH) + \beta_L \ln(PL) + \beta_K \ln(PK) + (1 - \beta_L - \beta_K) \ln(PF)$$

Thus, the coefficient for ln(PF) is $\hat{\beta}_F = 1 - \hat{\beta}_L - \hat{\beta}_K = 1 - 0.14521 - 0.164958 = 0.6898$

The standard error is calculated as follows:

$$\begin{aligned}\text{Var}(1 - \hat{\beta}_L - \hat{\beta}_K) &= \text{Var}(-1(\hat{\beta}_L + \hat{\beta}_K)) \\ &= \text{Var}(\hat{\beta}_L + \hat{\beta}_K) \\ &= \text{Var}(\hat{\beta}_L) + \text{Var}(\hat{\beta}_K) + 2\,\text{Cov}(\hat{\beta}_L, \hat{\beta}_K) \\ &= 0.011888 + 0.009611 + 2*(-0.007568) = 0.006363\end{aligned}$$

$$\text{se}(\hat{\beta}_F) = \sqrt{0.00636} = 0.0798$$

Next, following Berndt (1991) we calculate the returns-to-scale as follows:

$$r = \frac{1}{\partial \ln c / \partial \ln y} = \frac{1}{\hat{\beta}_Y} = 1.21163$$

This result implies an increasing returns-to-scale. In addition, we have a positive economies-of-scale ($e = r - 1 = 0.12$).

5. Augmented Nerlove Model

To allow the returns-to-scale to vary with the output, we add the quadratic output term into the estimation. That is, we estimate:

$$\ln c = \beta_0 + \beta_Y \ln y + \beta_{YY} \ln ysq + \beta_L \ln pl + \beta_K \ln pk$$

where *lnysq* : $[\ln(\text{KWH})]^2 = (\ln y)^2$

The estimation yields:

Source	SS	df	MS
Model	194.89936	4	48.7248401
Residual	1.77935149	94	.018929271
Total	196.678712	98	2.00692563

Number of obs	99
F(4, 94)	2574.05
Prob > F	0.0000
R-squared	0.9910
Adj R-squared	0.9906
Root MSE	.13758

lnc	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lny	.3013818	.0467794	6.44	0.000	.2085003	.3942633
lnysq	.0367468	.0032301	11.38	0.000	.0303334	.0431602
lnpl	.294079	.0722921	4.07	0.000	.1505413	.4376167
lnpk	.0507267	.0647092	0.78	0.435	-.077755	.1792084
_cons	-5.753319	.1703232	-33.78	0.000	-6.0915	-5.415138

The result shows that only ln *pk* is not significant at the 5 percent level, i.e. with *t*-statistic 0.78 and *p*-value 0.435. Meanwhile, the coefficient for the quadratic output term is significantly different from zero with *t*-statistic 11.38 and *p*-value 0.0000.[5]

Since *KWH* is not divided by *PF*, then the derived coefficient for ln(*PF*) and its standard deviation can be calculated with the same formula as in Cobb-Douglas, that is, $\hat{\beta}_F = 1 - \hat{\beta}_L - \hat{\beta}_K = 0.65519$ with se ($\hat{\beta}_F$) = $\sqrt{0.002715} = 0.0521$.

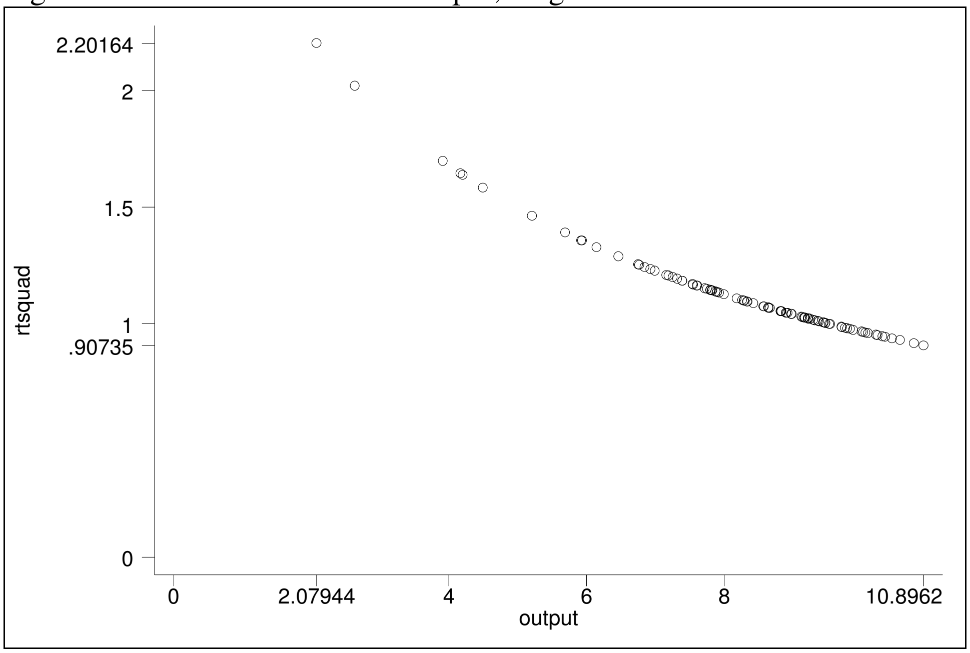
Now the returns-to-scale is calculated as:

$$r = \frac{1}{\partial \ln c / \partial \ln y} = \frac{1}{\hat{\beta}_Y + 2 * \hat{\beta}_{YY} \ln y}$$

[5] This is the same as testing for $H_0 : \beta_{YY} = 0$ using *F*-stat (see appendix) that gives F(1,94) = 129.42 with Prob > F = 0.0000, meaning that the null hypothesis is rejected.

Therefore, the returns-to-scale varies with the output level. We can plot this implied returns-to-scale as a function of output as in the following graph:

Figure 1. Returns-to-Scale and Output, Augmented Nerlove Model



Note: *rtsquad*: returns-to-scale for model with quadratic output term.

The graph implies that substantial scale economies are obtained at low levels of output. This is consistent with Christensen and Greene's finding.

6. Christensen and Greene's Model A

Finally we estimate the cost function using Christensen and Greenes' Model A. They used the following translog cost function:

$$\ln C = \alpha_0 + \alpha_Y \ln Y + \tfrac{1}{2}\gamma_{YY}(\ln Y)^2 + \sum_i \alpha_i \ln P_i + \tfrac{1}{2}\sum_i \sum_j \gamma_{ij} \ln P_i P_j + \sum_i \gamma_{Yi} \ln Y \ln P_i$$

where $\gamma_{ij} = \gamma_{ji}$, C is total cost, Y is output, and the P_i's are the prices of the factor inputs[6].

[6] The relationships among the parameters are such that the cost function is homogenous of degree one in prices. That is, $\sum_i \alpha_i = 1$,

$$\sum_i \gamma_{Yi} = 0,$$

$$\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = \sum_i \sum_j \gamma_{ij} = 0$$

For our estimation we adjust this function into:

$$\ln c = \beta_0 + \beta_Y \ln y + \beta_{YY} (\ln y)^2 + \beta_L \ln pl + \beta_{LL} (\ln pl)^2 + \beta_K \ln pk + \beta_{KK} (\ln pk)^2$$

$$+ \beta_{LK} (\ln pl)(\ln pk) + \beta_{LY} (\ln pl)(\ln y) + \beta_{KY} (\ln pk)(\ln y)$$

We obtain the estimation result as follows:

Source	SS	df	MS
Model	195.090039	9	21.676671
Residual	1.58867291	89	.017850257
Total	196.678712	98	2.00692563

Number of obs	= 99
F(9, 89)	= 1214.36
Prob > F	= 0.0000
R-squared	= 0.9919
Adj R-squared	= 0.9911
Root MSE	= .1336

lnc	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lny	.4541942	.0723514	6.28	0.000	.3104335	.597955
lnysq	.0319142	.0035916	8.89	0.000	.0247778	.0390505
lnpl	.1267841	.3339359	0.38	0.705	-.5367393	.7903076
lnplsq	.2975173	.2547844	1.17	0.246	-.208734	.8037685
lnpk	1.030376	.3827221	2.69	0.008	.2699156	1.790837
lnpksq	.486136	.2218225	2.19	0.031	.0453795	.9268925
lnplpk	-.8154519	.4190768	-1.95	0.055	-1.648149	.0172448
lnply	.0384884	.0308738	1.25	0.216	-.0228572	.099834
lnpky	-.1259864	.0399498	-3.15	0.002	-.2053658	-.0466071
_cons	-6.744742	.4174318	-16.16	0.000	-7.57417	-5.915314

It is shown that ln *pl*, $(\ln pl)^2$, (ln *pl*)(ln *pk*), and (ln *pl*)(ln *y*), that is, all ln *pl* and its interaction terms, are not significant at the 5 percent level. The *t*-statistics for these variables are 0.38, 1.17, -1.95, and 1.25, respectively, with *p*-values of 0.705, 0.246, 0.055, and 0.216.

The translog cost function used by Christensen and Greene does not constrain the structure of production to be homothetic, nor does it impose restrictions on the elasticities-of-substitution. However, these can be tested statistically. That is, homotheticity restriction is such that $\gamma_{Yi} = 0$ and homogeneity restriction is such that $\gamma_{Yi} = 0$ and $\gamma_{YY} = 0$.

Taking this into our estimation, we can examine the homotheticity in output by testing whether or not the coefficients of (ln *pl*)(ln *y*) and (ln *pk*)(ln *y*) are both zero. Our test gives an *F*-stat of 5.05 with *p*-value of 0.0084. Therefore, we reject the hypothesis at the 5 percent level. In other words, the cost function applied to 1970 data does not exhibit homotheticity.

On the other hand, the assessment of homogeneity in output involves the testing of whether the above restrictions hold (the coefficients $\hat{\beta}_{LY}$ and $\hat{\beta}_{KY}$ are both zero) and the coefficient of $(\ln y)^2$, $\hat{\beta}_{YY}$ is also zero. The test gives an *F*-stat of 44.99 with *p*-value of

0.0000. So, we reject the hypothesis for homogeneity, too. This is consistent with what Christensen and Greene found.

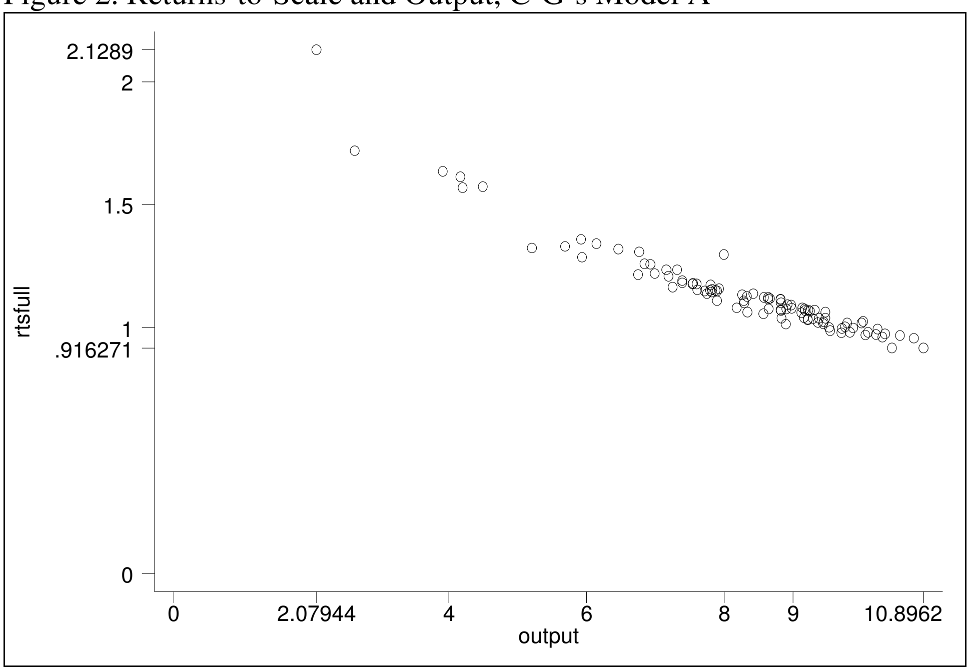
The fact that we reject both the hypothesis of homotheticity and the hypothesis of homogeneity implies that a model that allows non-homotheticity and non-homogeneity in output is required to represent the structure of productions for U.S. firms generating electric power.[7]

Just like in the previous two estimations, we are also interested in the returns-to-scale. Here it is calculated as follows:

$$r = \frac{1}{\partial \ln c / \partial \ln y} = \frac{1}{\hat{\beta}_Y + 2 * \hat{\beta}_{YY} \ln y + \hat{\beta}_{LY} \ln pl + \hat{\beta}_{KY} \ln pk}$$

Using this formula and plotting the values against the output gives a pattern depicted in Figure 2. Again, the graph tells us that in general, the electric utility firms exhibit an increasing returns-to-scale (r is greater than one), while the economies-of-scale[8] falls with output level. However, this economies-of-scale is exhausted at output levels of around 13,000 million kwh (that is, at lnkwh ≈ 9.5 on the graph).

Figure 2. Returns-to-Scale and Output, C-G's Model A



Note: rtsfull: returns-to-scale for full model, i.e. Model A.

[7] Clearly, the Cobb-Douglas function in Section 2 is a limiting form of this generalization in which we implicitly impose homotheticity and homogeneity.
[8] Here, following Christensen and Greene is defined as $e = 1 - 1/r$. However, we should note that $2 * \hat{\beta}_{YY} = \gamma_{YY}$